TEMPERATURE STRESSES IN A PLATE AND CHOICE OF OPTIMUM HEATING REGIME

A. A. Shevelev

Inzhenerno-fizicheskii zhurnal, Vol. 8, No. 1, pp. 79-81, 1965

Results are given of an investigation of the thermal stresses in an infinite plate for a problem of the third kind, when the temperature of the medium varies exponentially with time.

In a number of cases it is necessary to investigate the symmetrical heating of a plate, taking into account thermal stresses, under boundary conditions of the third kind, when the temperature of the medium varies exponentially with time, i.e.,

$$-\frac{\partial t(R\,\tau)}{\partial x}+\frac{\alpha}{\lambda}[t_m-(t_m-t_0)\exp(-k\tau)-t(R,\tau)]=0. \tag{1}$$

Here it is required to determine either the quantities defining the thermal effect of the medium on the heated body, or, if they are given, the maximum thermal stresses. This paper therefore establishes the connection between the maximum thermal stresses and Biot and Predvoditelev numbers.

The thermal stresses in a plate, when the origin of coordinates is located on one of its surfaces, are given by the expression [1]:

$$\frac{(1-\nu)\sigma_y}{\beta E}=\frac{(1-\nu)\sigma_z}{\beta E}=-t(x,\tau)+H_c+H_b\left(\frac{x}{R}-\frac{1}{2}\right),$$

$$H_c=\frac{1}{R}\int_0^R t(x,\tau)\,dx,\quad H_b=\frac{12}{R}\int_0^R\left(\frac{x}{R}-\frac{1}{2}\right)t(x,\tau)\,dx. \tag{2}$$

The variation of temperature for the boundary conditions indicated is given by the equation [2]:

$$t(x,\tau)=t_0+(t_m-t_0)[1-\cos\sqrt{\mathrm{Pd}}\,xR^{-1}(\cos\sqrt{\mathrm{Pd}}-\mathrm{Bi}^{-1}\sqrt{\mathrm{Pd}}\sin\sqrt{\mathrm{Pd}})^{-1}\times$$

$$\times\exp(-\mathrm{Pd\,Fo})-\sum_{n=1}^{\infty}\frac{A_n}{1-\mu_n^2/\mathrm{Pd}}\cos\mu_n\frac{x}{R}\exp(-\mu_n^2\,\mathrm{Fo})]. \tag{3}$$

When the plate is restrained from bending, i.e., when the coefficient H_b in (2) is zero, the thermal stresses are proportional to the difference between the temperature at any point and the mean. Integrating (2) and taking into account (3), we may write this difference in the form

$$\frac{(1-\nu)\sigma_y}{\beta E}=\frac{(1-\nu)\sigma_z}{\beta E}=-(t_m-t_0)\times$$

$$\times\left\{\sin\sqrt{\mathrm{Pd}}-\sqrt{\mathrm{Pd}}\cos\sqrt{\mathrm{Pd}}\frac{x}{R}\left[\sqrt{\mathrm{Pd}}\left(\cos\sqrt{\mathrm{Pd}}-\frac{\sqrt{\mathrm{Pd}}}{\mathrm{Bi}}\sin\sqrt{\mathrm{Pd}}\right)\right]^{-1}\times\right.$$

$$\left.\times\exp(-\mathrm{Pd\,Fo})+\sum_{n=1}^{\infty}\frac{A_n}{1-\mu_n^2/\mathrm{Pd}}\left(\frac{\sin\mu_n}{\mu_n}-\cos\mu_n\frac{x}{R}\right)\exp(-\mu_n^2\,\mathrm{Fo})\right\}. \tag{4}$$

It follows from (4) that, when the plate is cooled, the greatest tensile stresses will be at the surface x = R, and the greatest compressive stresses at x = 0. When there is heating, the stresses change sign. For both heating and cooling, the absolute value of the thermal stresses at points x = R will be approximately twice as large as at points x = 0.

Since we are interested in practice in the maximum stress, we have investigated the stresses at x = R. The curves of Fig. 1a show the relationship between the Bi and Pd numbers and the maximum stress, calculated according to (4) using three terms of the series, and correct to five places. Clearly, the thermal stresses increase with increases in Bi and Pd. When these parameters become infinite, the relative value of the thermal stresses is unity, i.e., the stresses are determined by the difference between the initial body temperature and the maximum temperature of the medium.

In Fig. 1b the Fourier number corresponding to the time of occurrence of maximum stress is shown as a function of Bi and Pd. It may be necessary to use this diagram to decide the permissible thermal stress in plates subjected to other variable loads as well as those due to heating, e.g., the increase in pressure in the housing of a steam turbine during start-up.

It is clear from the figure that, when heating at the peak rate, i.e., when Bi = ∞, while the temperature of the medium varies, the maximum stress does not develop at once, but depends on the Pd number.

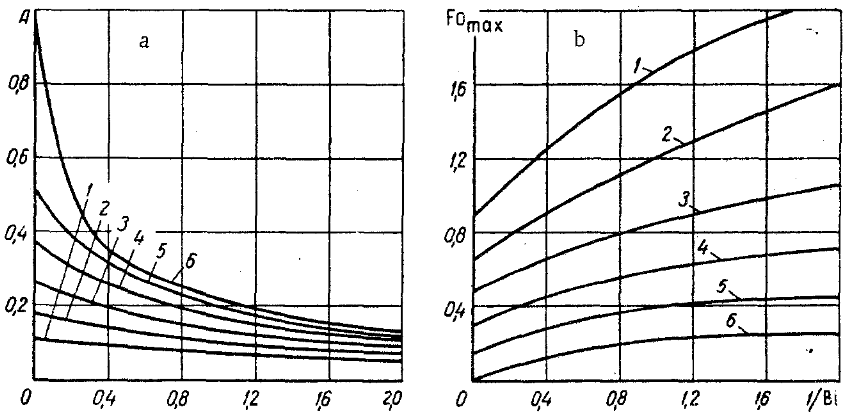

Fig. 1. Dependence of $\frac{(1-\nu)\sigma_{max}}{\beta E(t_m - t_0)} \equiv A$ (a) and Fo_{max} (b) on Bi and Pd:

1 – Pd = 0.5; 2 – 1; 3 – 2; 4 – 4; 5 – 10; 6 – ∞

The application of the relationships obtained may be illustrated by considering an example.

Let the permissible value of the thermal stresses (σ_{max}) be 19,600 newton/cm^2, the coefficient of linear expansion $\beta = 12.5 \cdot 10^{-6}$ /degree, the modulus of elasticity $E = 19.62 \cdot 10^6$ n/cm^2, the Poisson ratio $\nu = 0.3$, the maximum difference between the temperature of the medium and initial body temperature $t_m - t_0 = 287$°C, the thermal diffusivity $a = 0.045$ m^2/hr, Bi = 2.5, plate thickness R = 0.3 m. We shall determine the optimum behavior of the temperature of the medium during heating, and the time of occurrence of the maximum stress.

For the given conditions, $(1-\nu)\sigma_{max}/\beta E(t_m - t_0)$ is 0.195. Corresponding to this value of $(1-\nu)\sigma_{max}/$ $/\beta E(t_m - t_0)$ from Fig. 1 we have Pd = 2.0, $Fo_{max} = 0.66$. Consequently, the optimum increase in the temperature of the medium is given by:

$$t_c(\tau) = t_m - (t_m - t_0)\exp(-\mathrm{Pd\,Fo}) = 307 - 287\exp(-4\tau),$$

and the time of occurrence of the maximum stress is

$$\tau_{max} = \frac{Fo_{max} R^2}{a} = 1.32 \text{ hr.}$$

There will be analogous cases where it is required to find the permissible value of the Biot number for a given maximum stress and Pd number, or where it is required to check the stresses in conditions when Bi and Pd numbers are known.

NOTATION

$Bi = \alpha R/\lambda$ – Biot number, $Pd = kR^2/a$ – Predvoditelev number, $Fo = a\tau/R^2$ – Fourier number, α – heat transfer coefficient, k – constant, a – thermal diffusivity, λ – thermal conductivity, τ – time, β – coefficient of linear expansion, E – modulus of elasticity, t_0 – initial temperature of body, t_m – maximum temperature of medium.

REFERENCES

1. B. C. Gatewood, Thermal Stresses [Russian translation], IL, 1959.
2. A. V. Lykov, Theory of Heat Conduction [in Russian], GITTL, 1952.

7 April 1964

Lenin Polytechnic Institute, Kharkov.